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A note on heat and mass transfer from a sphere in Stokes flow at low Péclet number

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Abstract

We consider the low Péclet number, Pe $\ll 1$, asymptotic solution for steady-state heat and mass transfer from a sphere immersed in Stokes flow with a Robin boundary condition on its surface, representing Newton cooling or a first-order chemical reaction. The application of van Dyke's rule up to terms of $O(\mathrm{Pe}^3)$ shows that the $O(\mathrm{Pe}^3 \log \mathrm{Pe})$ terms in the expression for the average Nusselt/Sherwood number are double those previously derived in the literature. Inclusion of the $O(\mathrm{Pe}^3)$ terms is shown to increase significantly the range of validity of the expansion.

Keywords: Péclet, Stokes, heat transfer, mass transfer, sphere

1. Introduction

In 1962, Acrivos and Taylor [1] derived an asymptotic expansion in terms of low Péclet number, Pe $\ll 1$, for steady heat and mass transfer from an isothermal sphere in Stokes flow. They gave a simple expression for the average Nusselt (Sherwood) number, which is an important measure of the rate of heat (mass) transfer from the sphere. Subsequently, Gupalo and Ryazantsev [2] considered a Robin boundary condition on the sphere (representing Newton cooling or a first-order chemical reaction), and low Reynolds number (Re $\ll 1$) corrections to Stokes flow, such that the Schmidt number Sc $=$ Pe/Re $= O(1)$. In both cases, they constructed the $O(\mathrm{Pe}^3 \log \mathrm{Pe})$ terms in the inner solution without calculating the $O(\mathrm{Pe}^3)$ terms. However, truncation of Van Dyke's matching process at terms including $\log \mathrm{Pe}$ can lead to incorrect results (cf. Hinch [3]). Rather, truncation should occur at terms of integer order in Pe. Here we determine the $O(\mathrm{Pe}^3 \log \mathrm{Pe})$ terms for Stokes flow (Re $= 0$) by matching up to $O(\mathrm{Pe}^3)$. Comparison to numerics shows that inclusion of the extra $O(\mathrm{Pe}^3)$ terms extends the validity of the expression for the average Nusselt/Sherwood number to a substantially larger range of Péclet numbers.

2. Theory

We consider steady convective-diffusive transport around a sphere [1, 2]:

$$\nabla^2 h = \mathrm{Pe}\, \boldsymbol{u} \cdot \nabla h, \tag{1}$$

where h represents either concentration (mass transfer) or temperature (heat transfer). The

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boundary conditions are (cf. Gupalo and Ryazantsev [2]):

$$\frac{\partial h}{\partial r} = \beta(h-1), \quad \text{on } r=1; \quad h \to 0, \quad \text{as } r \to \infty. \tag{2}$$

The non-dimensional fluid flow-field, $\boldsymbol{u} = u_r \boldsymbol{e}_r + u_\theta \boldsymbol{e}_\theta$, is Stokes flow [1]:

$$u_r = \left(1 - \frac{3}{2r} + \frac{1}{2r^3}\right)\mu; \quad u_\theta = -\left(1 - \frac{3}{4r} - \frac{1}{4r^3}\right)(1-\mu^2)^{\frac{1}{2}}, \tag{3}$$

where $\mu = \cos\theta$. For Pe $\ll 1$, this is a singular perturbation problem (cf. [1, 2]), and we write the inner and outer solutions, h and H respectively, as perturbation expansions in Pe:

$$h(r,\,\mu) = \sum_{n=0}^{\infty} \mathrm{Pe}^n h_n(r,\,\mu); \quad H(\rho,\,\mu) = \sum_{n=0}^{\infty} \mathrm{Pe}^{n+1} H_n(\rho,\,\mu), \tag{4}$$

with inner coordinate system $(r,\,\mu)$ and outer coordinate system $(\rho,\,\mu)$, where $\rho = r\mathrm{Pe}$. The first three terms of the inner solution were derived in [2] to be (letting $\mathrm{Sc} = \mathrm{Pe}/\mathrm{Re} \to \infty$):

$$h_0 = \frac{\beta}{(1+\beta)r}, \tag{5}$$

$$h_1 = \frac{\beta}{\beta+1}\left\{-\frac{1}{2} + \frac{\beta}{2(\beta+1)r}\right\} + \frac{\beta}{1+\beta}\left\{\frac{1}{2} - \frac{3}{4r} + \frac{3(\beta+3)}{8(\beta+2)r^2} - \frac{1}{8r^3}\right\}\mu, \tag{6}$$

$$h_2 = \frac{\beta}{1+\beta}\left(\sum_{k=0}^{2}\left(a_2^{(k)} r^k + b_2^{(k)} r^{-k-1}\right)P_k(\mu) + \sum_{k=0}^{2} h_2^{(k)}(r) P_k(\mu)\right). \tag{7}$$

In expression (7), $P_k(\mu)$ are Legendre polynomials [4] and the functions $h_2^{(k)}(r)$ are given by:

$$h_2^{(0)}(r) = \frac{r}{6} - \frac{\log r}{2} + \frac{7\beta+23}{96(\beta+2)r^2} + \frac{1}{48r^3} - \frac{\beta+3}{64(\beta+2)r^4} + \frac{1}{240r^5}, \tag{8a}$$

$$h_2^{(1)}(r) = \frac{\beta}{1+\beta}\left\{\frac{1}{4} - \frac{3}{8r} - \frac{1}{16r^3}\right\}, \tag{8b}$$

$$h_2^{(2)}(r) = \frac{r}{12} - \frac{5}{24} + \frac{5\beta+12}{16(\beta+2)r} - \frac{5(13\beta+35)}{192(\beta+2)r^2} + \frac{\log r}{16r^3} - \frac{\beta+3}{32(\beta+2)r^4} + \frac{5}{672r^5}. \tag{8c}$$

The constants $a_2^{(k)}$ and $b_2^{(k)}$ in (7) are given by (where γ is Euler's constant):

$$a_2^{(0)} = -\frac{\log \mathrm{Pe}}{2} - \frac{\gamma}{2} + \frac{\beta+3}{8(\beta+1)}; \quad a_2^{(1)} = -\frac{1}{4}; \quad a_2^{(2)} = 0; \tag{9a}$$

$$b_2^{(0)} = \frac{\beta \log \mathrm{Pe}}{2(\beta+1)} + \frac{\gamma\beta}{2(\beta+1)} - \frac{359}{960} - \frac{1}{32(\beta+2)} - \frac{203}{480(\beta+1)} + \frac{1}{4(\beta+1)^2}; \tag{9b}$$

$$b_2^{(1)} = \frac{7}{16} - \frac{3}{8(\beta+2)} - \frac{3}{8(\beta+1)}; \quad b_2^{(2)} = \frac{235}{1344} + \frac{3}{16(\beta+2)} + \frac{3}{112(\beta+3)}. \tag{9c}$$

The first two terms of the outer expansion are (letting $\mathrm{Sc} \to \infty$ in the solutions in [2]) :

$$H_0 = \frac{\beta}{(1+\beta)\rho} e^{\frac{1}{2}\rho(\mu-1)}, \tag{10}$$

$$H_1 = e^{\frac{1}{2}\rho\mu}\left(\frac{\beta}{1+\beta}\right)\left\{\left(\frac{\pi}{\rho}\right)^{\frac{1}{2}} \sum_{k=0}^{2} A_1^{(k)} K_{k+\frac{1}{2}}\left(\frac{\rho}{2}\right) P_k(\mu) + \sum_{k=0}^{2} G_1^{(k)}(\rho) P_k(\mu)\right\}, \tag{11}$$

where $K_{k+\frac{1}{2}}(\cdot)$ is a modified Bessel function [4], and the constants $A_1^{(k)}$ in (11) are given by:

$$A_1^{(0)} = \frac{\beta}{2\pi(\beta+1)} + \frac{\gamma}{2\pi}; \quad A_1^{(1)} = -\frac{3(\gamma-1)}{4\pi}; \quad A_1^{(2)} = \frac{\gamma+\log 2-3}{4\pi}. \tag{12}$$

The functions $G_1^{(k)}$ in (11) are given by:

$$G_1^{(0)}(\rho) = \frac{e^{-\frac{\rho}{2}}}{2\rho}\Big(\log\rho + e^{\rho}\Gamma[0,\,\rho]\Big), \tag{13a}$$

$$G_1^{(1)}(\rho) = -\frac{3e^{-\frac{\rho}{2}}}{4\rho^2}\Big(2+(\rho+2)\log\rho-(\rho-2)e^{\rho}\Gamma[0,\,\rho]\Big), \tag{13b}$$

$$G_1^{(2)}(\rho) = \frac{e^{-\frac{\rho}{2}}}{4\rho^3}\Big(6(\rho+6)+(\rho^2+6\rho+12)\log\left(\frac{\rho}{2}\right)+(\rho^2-6\rho+12)e^{\rho}\Gamma[0,\,\rho]\Big), \tag{13c}$$

where $\Gamma[0,\,\rho]$ is the incomplete gamma function [4]. We remark that it was simpler to re-derive H_1 in expression (11) for Stokes flow rather than letting $\mathrm{Sc}\to\infty$ in the solutions in [2].

2.1. The outer solution H_2

We have derived the solution for the next term, H_2, in the outer expansion to be:

$$H_2 = e^{\frac{1}{2}\rho\mu}\left(\frac{\beta}{1+\beta}\right)\left[\left(\frac{\pi}{\rho}\right)^{\frac{1}{2}}\sum_{k=0}^{1}A_2^{(k)}K_{k+\frac{1}{2}}\left(\frac{\rho}{2}\right)P_k(\mu)+\sum_{k=0}^{4}G_2^{(k)}(\rho)P_k(\mu)\right]. \tag{14}$$

Matching $\sum_{n=0}^{2}\mathrm{Pe}^{n+1}H_n$ to $\sum_{n=0}^{2}\mathrm{Pe}^{n}h_n$ using van Dyke's rule determined the constants $A_2^{(k)}$:

$$A_2^{(0)} = \frac{\beta(\log\mathrm{Pe}+\gamma)}{2\pi(\beta+1)} - \frac{419\beta^3+2172\beta^2+3253\beta+1260}{960\pi(\beta+1)^2(\beta+2)}, \quad A_2^{(1)} = \frac{3(\beta+3)}{16\pi(\beta+2)}. \tag{15}$$

The functions $G_2^{(k)}(\rho)$ in expression (14) are given by:

$$G_2^{(k)}(\rho) = -\frac{1}{\rho^{\frac{1}{2}}}K_{k+\frac{1}{2}}\left(\frac{\rho}{2}\right)\int_0^{\rho}u^{\frac{3}{2}}I_{k+\frac{1}{2}}\left(\frac{u}{2}\right)R_k(u)\,\mathrm{d}u - \frac{1}{\rho^{\frac{1}{2}}}I_{k+\frac{1}{2}}\left(\frac{\rho}{2}\right)\int_{\rho}^{\infty}u^{\frac{3}{2}}K_{k+\frac{1}{2}}\left(\frac{u}{2}\right)R_k(u)\,\mathrm{d}u, \tag{16}$$

where

$$R_0(\rho) = \frac{9e^{-\frac{\rho}{2}}}{20\rho^4}\left\{2\rho+\frac{9-\beta}{18(\beta+1)}\rho^2-\big(\rho^2+\rho+2\big)\left(\log\rho+\gamma\right)-\big(\rho^2-\rho+2\big)\,e^{\rho}\Gamma(0,\rho)\right\}, \tag{17a}$$

$$R_1(\rho) = \frac{9e^{-\frac{\rho}{2}}}{40\rho^5}\left\{-12\rho+\frac{\beta-9}{3(\beta+1)}\rho^2-\frac{4\beta+9}{3(\beta+1)}\rho^3+\big(4\rho^3+9\rho^2+6\rho+12\big)\left(\log\rho+\gamma\right) - \big(4\rho^3-9\rho^2+6\rho-12\big)\,e^{\rho}\Gamma(0,\rho)\right\}, \tag{17b}$$

$$R_2(\rho) = \frac{9e^{-\frac{\rho}{2}}}{56\rho^4}\Bigg\{27\rho + \frac{38\beta+45}{9(\beta+1)}\rho^2 - \left(4\rho^2+17\rho+34\right)(\log\rho+\gamma) - \left(4\rho^2-17\rho+34\right)e^{\rho}\Gamma(0,\rho)\Bigg\}, \quad (17c)$$

$$R_3(\rho) = \frac{9e^{-\frac{\rho}{2}}}{40\rho^5}\Bigg\{-48\rho - 12\rho^2 - 2\rho^3 + \left(\rho^3+6\rho^2+24\rho+48\right)(\log\rho+\gamma) - \left(\rho^3-6\rho^2+24\rho-48\right)e^{\rho}\Gamma(0,\rho)\Bigg\}, \quad (17d)$$

$$R_4(\rho) = \frac{9e^{-\frac{\rho}{2}}}{280\rho^4}\Bigg\{12\rho + 3\rho^2 - \left(\rho^2+6\rho+12\right)(\log\rho+\gamma) - \left(\rho^2-6\rho+12\right)e^{\rho}\Gamma(0,\rho)\Bigg\}. \quad (17e)$$

2.2. The inner solution h_3

The next term in the inner solution, h_3, is then given by:

$$h_3 = \left(\frac{\beta}{1+\beta}\right)\sum_{k=0}^{2} a_3^{(k)}\left(\frac{k-\beta}{k+1+\beta}r^{-k-1} + r^k\right)P_k(\mu) + \left(\frac{\beta}{1+\beta}\right)\sum_{k=0}^{3}\Bigg\{\left(\frac{\mathrm{d}h_3^{(k)}/\mathrm{d}r(1) - h_3^{(k)}(1)\beta}{k+1+\beta}\right)r^{-k-1} + h_3^{(k)}(r)\Bigg\}P_k(\mu), \quad (18)$$

where the functions $h_3^{(k)}(r)$ are the r-components of the particular solutions given by:

$$h_3^{(0)}(r) = -\frac{r^2}{24} + \frac{(5\beta+3)r}{24(\beta+1)} - \frac{\beta\log r}{4(\beta+1)} + \frac{19\beta^2+23\beta-12}{192(\beta+1)(\beta+2)r^2} + \frac{\beta}{96(\beta+1)r^3} - \frac{7\beta^2+9\beta-4}{384(\beta+1)(\beta+2)r^4} + \frac{\beta}{480(\beta+1)r^5}, \quad (19a)$$

$$\begin{aligned} h_3^{(1)}(r) = {} & \frac{\beta\left(4r^3-6r^2-1\right)\log \mathrm{Pe}}{16(\beta+1)r^3} + \frac{3r^2}{40} + \frac{r}{8} + \frac{\beta\gamma}{\beta+1}\left(-\frac{1}{16r^3} - \frac{3}{8r} + \frac{1}{4}\right) \\ & + \left(-\frac{1}{320r^5} + \frac{9}{640r^3} + \frac{23\beta+1}{640(\beta+2)r^2} - \frac{3r}{8}\right)\log r \\ & - \frac{1499\beta^3+6528\beta^2+8725\beta+3456}{1920(\beta+1)^2(\beta+2)} + \frac{551\beta^3+2844\beta^2+4201\beta+1668}{1280(\beta+1)^2(\beta+2)r} \\ & + \frac{23\beta+1}{1920(\beta+2)r^2} + \frac{5237\beta^4+42471\beta^3+118927\beta^2+132033\beta+46980}{53760(\beta+1)^2(\beta+2)(\beta+3)r^3} \\ & - \frac{3(\beta+23)}{8960(\beta+2)r^4} - \frac{235\beta^2+1463\beta+2238}{26880(\beta+2)(\beta+3)r^5} + \frac{3(\beta+3)}{1280(\beta+2)r^6} - \frac{9}{17920r^7}, \end{aligned} \quad (19b)$$

$$h_3^{(2)}(r) = \frac{\beta}{\beta+1}\Bigg\{\frac{\log r}{32r^3} + \frac{(\beta-3)r}{96\beta} - \frac{5}{48} + \frac{25\beta^2+42\beta-4}{96\beta(\beta+2)r}$$

$$- \frac{5\left(25\beta^2+35\beta-12\right)}{384\beta(\beta+2)r^2} + \frac{1}{160r^3} - \frac{7\beta^2+9\beta-4}{192\beta(\beta+2)r^4} + \frac{5}{1344r^5}\Bigg\}, \quad (19c)$$

$$h_3^{(3)}(r) = \left(-\frac{3}{640r^5} + \frac{9(\beta+3)}{896(\beta+2)r^4} - \frac{3}{80r^3} + \frac{3}{160r^2}\right)\log r + \frac{r^2}{120} - \frac{r}{40} + \frac{(10\beta+23)}{160(\beta+2)}$$
$$- \frac{38\beta+97}{320(\beta+2)r} + \frac{1261\beta^2+7448\beta+10923}{8960(\beta+2)(\beta+3)r^2} - \frac{3\left(31\beta^2+210\beta+345\right)}{1120(\beta+2)(\beta+3)r^3}$$
$$+ \frac{9(\beta+3)}{6272(\beta+2)r^4} - \frac{395\beta^2+2551\beta+4026}{35840(\beta+2)(\beta+3)r^5} + \frac{(\beta+3)}{640(\beta+2)r^6} - \frac{1}{3360r^7}. \quad (19d)$$

Matching $\sum_{n=0}^{3} \mathrm{Pe}^n h_n$ to $\sum_{n=0}^{2} \mathrm{Pe}^{n+1} H_n$ using van Dyke's rule determines the constants $a_3^{(k)}$ to be:

$$a_3^{(0)} = -\frac{\beta(2\log \mathrm{Pe}+\gamma)}{4(\beta+1)} - \frac{9\mathcal{I}_1}{20} - \frac{(9-\beta)\mathcal{I}_2}{40(1+\beta)} + \frac{779\beta^3+3252\beta^2+3973\beta+1260}{1920(\beta+1)^2(\beta+2)}, \quad (20a)$$

$$a_3^{(1)} = -\frac{3\log \mathrm{Pe}}{8} + \frac{(6\beta+11)}{40(\beta+1)} - \frac{3\gamma}{8}; \quad a_3^{(2)} = -\frac{1}{24}. \quad (20b)$$

In (20a), the constants $\mathcal{I}_1 \approx -1.1063$ and $\mathcal{I}_2 \approx -0.5772$ are calculated from the integrals:

$$\mathcal{I}_1 = \int_0^1 \frac{e^{-u}}{u^3}\left\{2u - (2+u+u^2)(\gamma+\log u) - (u^2-u+2)e^u\Gamma(0,\, u)\right\} + \frac{1}{2u}\,\mathrm{d}u$$
$$+ \int_1^\infty \frac{e^{-u}}{u^3}\left\{2u - (2+u+u^2)(\gamma+\log u) - (u^2-u+2)e^u\Gamma(0,\, u)\right\}\mathrm{d}u, \quad (21a)$$

$$\mathcal{I}_2 = \int_0^1 \left(\frac{e^{-u}-1}{u}\right)\mathrm{d}u + \int_1^\infty \frac{e^{-u}}{u}\,\mathrm{d}u. \quad (21b)$$

3. Results and discussion

The average Nusselt (Sherwood) number is an important measure of the rate of heat (mass) transfer from the surface of the sphere. It is defined by:

$$\overline{\mathrm{Nu}} = -\frac{1}{2}\int_{-1}^{1} \left.\frac{\partial h}{\partial r}\right|_{r=1} \mathrm{d}\mu, \quad (22)$$

and is equal to (cf. Gupalo and Ryazantsev [2] in the limit that $\mathrm{Sc} \to \infty$):

$$\overline{\mathrm{Nu}} = \frac{\beta}{1+\beta}\Bigg\{1 + \frac{\beta}{2(1+\beta)}\left(\mathrm{Pe}+\mathrm{Pe}^2\log\mathrm{Pe}\right) + \frac{\beta}{1+\beta}\left(\frac{\gamma}{2} + \frac{121\beta^2+168\beta-193}{960(\beta+1)(\beta+2)}\right)\mathrm{Pe}^2$$
$$+ \frac{\beta^2}{2(1+\beta)^2}\mathrm{Pe}^3\log\mathrm{Pe} + \frac{\beta}{1+\beta}\Bigg(\frac{9\mathcal{I}_1}{20} + \frac{(9-\beta)\mathcal{I}_2}{40(1+\beta)} + \frac{\gamma\beta}{4(\beta+1)}$$
$$- \frac{359\beta^3+1692\beta^2+2353\beta+900}{960(\beta+1)^2(\beta+2)}\Bigg)\mathrm{Pe}^3\Bigg\} + o(\mathrm{Pe}^3). \quad (23)$$

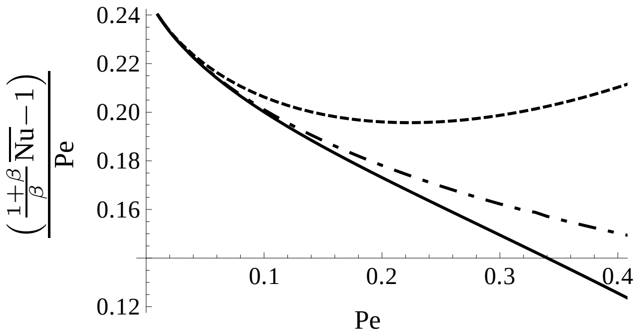

Figure 1: Plot of $\big((1+\beta)\overline{\text{Nu}}/\beta - 1\big)/\text{Pe}$ as a function of the Péclet number, $0.01 \leq \text{Pe} \leq 0.4$, with $\beta = 1$ calculated analytically from (23) (solid line) and numerically (dot-dashed line). The dashed line is the previous asymptotic approximation derived by Gupalo and Ryazantsev [2] (in the limit $\text{Sc} \to \infty$).

Letting $\beta \to \infty$ in (23) corresponds to constant temperature (concentration) on the sphere, so that $h = 1$ on $r = 1$, (cf. Acrivos and Taylor [1]), and gives:

$$\overline{\text{Nu}} = 1 + \frac{1}{2}\text{Pe} + \frac{1}{2}\text{Pe}^2 \log \text{Pe} + \text{Pe}^2 \left(\frac{\gamma}{2} + \frac{121}{960}\right) + \frac{1}{2}\text{Pe}^3 \log \text{Pe} + \text{Pe}^3 \left\{\frac{\gamma}{4} - \frac{359}{960} + \frac{9\mathcal{I}_1}{20} - \frac{\mathcal{I}_2}{40}\right\} + o(\text{Pe}^3). \quad (24)$$

In both expressions, (23) and (24), the coefficient of $\text{Pe}^3 \log \text{Pe}$ is twice that reported in [1] and [2]. Figure 1 shows a comparison of the analytical solution, (23), to numerical simulations for $\beta = 1$ as a function of Pe. As expected, this plot shows that the new asymptotic solution (23) is valid over a substantially increased range of Péclet numbers; the percentage error in the $O(\text{Pe})$ and higher order contributions to $(1+\beta)\overline{\text{Nu}}/\beta$ is within 8% for Pe up to 0.3.

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References

[1] A. Acrivos, T. D. Taylor, Heat and Mass Transfer from Single Spheres in Stokes Flow, Phys. Fluids 5 (1962) 387–394.

[2] Y. P. Gupalo, Y. S. Ryazantsev, Mass and heat transfer from a sphere in a laminar flow, Chem. Eng. Sci. 27 (1) (1972) 61–68.

[3] E. J. Hinch, Perturbation Methods, Cambridge University Press, Cambridge, 1991.

[4] M. Abramowitz, I. A. Stegun (Eds.), Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, National Bureau of Standards Applied Mathematics Series, 55, U.S. Goverment Printing Office, Washington, DC, 1964.

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