A note on heat and mass transfer from a sphere in Stokes flow at low Péclet number

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Abstract
We consider the low Péclet number, Pe ≪ 1, asymptotic solution for steady-state heat and mass transfer from a sphere immersed in Stokes flow with a Robin boundary condition on its surface, representing Newton cooling or a first-order chemical reaction. The application of van Dyke’s rule up to terms of O(Pe^3) shows that the O(Pe^3 log Pe) terms in the expression for the average Nusselt/Sherwood number are double those previously derived in the literature. Inclusion of the O(Pe^8) terms is shown to increase significantly the range of validity of the expansion.

Keywords: Péclet, Stokes, heat transfer, mass transfer, sphere

1. Introduction
In 1962, Acrivos and Taylor [1] derived an asymptotic expansion in terms of low Péclet number, Pe ≪ 1, for steady heat and mass transfer from an isothermal sphere in Stokes flow. They gave a simple expression for the average Nusselt (Sherwood) number, which is an important measure of the rate of heat (mass) transfer from the sphere. Subsequently, Gupalo and Ryazantsev [2] considered a Robin boundary condition on the sphere (representing Newton cooling or a first-order chemical reaction), and low Reynolds number (Re ≪ 1) corrections to Stokes flow, such that the Schmidt number Sc = Pe/Re = O(1). In both cases, they constructed the O(Pe^3 log Pe) terms in the inner solution without calculating the O(Pe^8) terms. However, truncation of Van Dyke’s matching process at terms including log Pe can lead to incorrect results (cf. Hinch [3]). Rather, truncation should occur at terms of integer order in Pe. Here we determine the O(Pe^3 log Pe) terms for Stokes flow (Re = 0) by matching up to O(Pe^8). Comparison to numerics shows that inclusion of the extra O(Pe^8) terms extends the validity of the expression for the average Nusselt/Sherwood number to a substantially larger range of Péclet numbers.

2. Theory
We consider steady convective-diffusive transport around a sphere [1, 2]:

∇^2 h = Pe u · ∇h,  \( \text{(1)} \)

where h represents either concentration (mass transfer) or temperature (heat transfer). The
boundary conditions are (cf. Gupalo and Ryazantsev [2]):

\[
\frac{\partial h}{\partial r} = \beta (h - 1), \quad \text{on } r = 1; \quad h \to 0, \quad \text{as } r \to \infty.
\]  

(2)

The non-dimensional fluid flow-field, \( \mathbf{u} = u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta \), is Stokes flow [1]:

\[
u_r = \left(1 - \frac{3}{2r} + \frac{1}{2r^3}\right) \mu; \quad \nu_\theta = -\left(1 - \frac{3}{4r} - \frac{1}{4r^3}\right) (1 - \mu^2)^{\frac{1}{2}},
\]  

(3)

where \( \mu = \cos \theta \). For \( \text{Pe} \ll 1 \), this is a singular perturbation problem (cf. [1, 2]), and we write the inner and outer solutions, \( h \) and \( H \) respectively, as perturbation expansions in \( \text{Pe} \):

\[
h(r, \mu) = \sum_{n=0}^{\infty} \text{Pe}^n h_n(r, \mu); \quad H(\rho, \mu) = \sum_{n=0}^{\infty} \text{Pe}^{n+1} H_n(\rho, \mu),
\]  

(4)

with inner coordinate system \((r, \mu)\) and outer coordinate system \((\rho, \mu)\), where \( \rho = r \text{Pe} \). The first three terms of the inner solution were derived in [2] to be (letting Sc = \( \text{Pe}/\text{Re} \to \infty \)):

\[
h_0 = \frac{\beta}{(1 + \beta)} r, \quad
\]  

\[
h_1 = \frac{\beta}{1 + \beta} \left\{ \frac{1}{2} \frac{1}{2(1 + \beta)} \right\} + \frac{\beta}{1 + \beta} \left\{ \frac{1}{2} \frac{3}{4r} + \frac{3(\beta + 3)}{8(\beta + 2)r^2} - \frac{1}{8r^3} \right\} \mu,
\]  

\[
h_2 = \frac{\beta}{1 + \beta} \left( \sum_{k=0}^{2} \left( a_2^{(k)} r^k + b_2^{(k)} r^{-k-1} \right) P_k(\mu) + \sum_{k=0}^{2} h_2^{(k)}(r) P_k(\mu) \right).
\]  

(7)

In expression (7), \( P_k(\mu) \) are Legendre polynomials [4] and the functions \( h_2^{(k)}(r) \) are given by:

\[
h_2^{(0)}(r) = \frac{r}{6} \left[ \frac{\log r}{2} + \frac{7\beta + 23}{96(\beta + 2)r^2} + \frac{1}{48r^3} - \frac{\beta + 3}{64(\beta + 2)r^4} + \frac{1}{240r^5} \right],
\]  

(8a)

\[
h_2^{(1)}(r) = \frac{\beta}{1 + \beta} \left\{ \frac{1}{4} \frac{3}{8r} - \frac{1}{8r^3} \right\},
\]  

(8b)

\[
h_2^{(2)}(r) = \frac{r}{12} \left[ \frac{5\beta + 12 + 5(13\beta + 35)}{16(\beta + 2)r^2} - \frac{\log r}{16r^3} - \frac{\beta + 3}{32(\beta + 2)r^4} + \frac{5}{672r^5} \right].
\]  

(8c)

The constants \( a_2^{(k)} \) and \( b_2^{(k)} \) in (7) are given by (where \( \gamma \) is Euler’s constant):

\[
a_2^{(0)} = -\frac{\log \text{Pe}}{2} \gamma + \frac{\beta + 3}{8(\beta + 1)}; \quad a_2^{(1)} = \frac{1}{4}, \quad a_2^{(2)} = 0.
\]  

(9a)

\[
b_2^{(0)} = \frac{\beta \log \text{Pe}}{2(\beta + 1)} + \frac{\gamma \beta}{2(\beta + 1)} \frac{359}{960} - \frac{1}{32(\beta + 2)} - \frac{203}{480(\beta + 1)} + \frac{1}{4(\beta + 1)^2};
\]  

\[
b_2^{(1)} = \frac{7}{16} - \frac{3}{8(\beta + 2)} - \frac{3}{8(\beta + 1)}; \quad b_2^{(2)} = \frac{235}{1344} + \frac{3}{16(\beta + 2)} + \frac{3}{112(\beta + 3)}.
\]  

(9b)

The first two terms of the outer expansion are (letting Sc \( \to \infty \) in the solutions in [2]):

\[
H_0 = \frac{\beta}{(1 + \beta)} e^\frac{\mu}{(1 + \beta^2)} e^{\frac{1}{2}(\mu - 1)},
\]  

(10)

\[
H_1 = e^{\frac{\mu}{(1 + \beta)}} \left\{ \frac{\beta}{(1 + \beta)} \sum_{k=0}^{2} \frac{A_1^{(k)} K_{k+\frac{1}{2}} \left( \frac{\mu}{2} \right) P_k(\mu) + \sum_{k=0}^{2} G_1^{(k)}(\mu) P_k(\mu) }{2} \right\},
\]  

(11)
where \( K_{k+\frac{1}{2}}(\cdot) \) is a modified Bessel function [4], and the constants \( A_1^{(k)} \) in (11) are given by:

\[
A_1^{(0)} = \frac{\beta}{2\pi(\beta + 1)} + \frac{\gamma}{2\pi}, \quad A_1^{(1)} = -\frac{3(\gamma - 1)}{4\pi}, \quad A_1^{(2)} = \frac{\gamma + \log 2 - 3}{4\pi}.
\] (12)

The functions \( G_1^{(k)} \) in (11) are given by:

\[
G_1^{(0)}(\rho) = \frac{e^{-\frac{\rho}{2}}}{2\rho} \left( \log \rho + e^{\rho}\Gamma[0, \rho] \right),
\] (13a)

\[
G_1^{(1)}(\rho) = -\frac{3e^{-\frac{\rho}{2}}}{4\rho^2} \left( 2 + (\rho + 2) \log \rho - (\rho - 2)e^{\rho}\Gamma[0, \rho] \right),
\] (13b)

\[
G_1^{(2)}(\rho) = \frac{e^{-\frac{\rho}{2}}}{4\rho^3} \left( 6(\rho + 6) + (\rho^2 + 6\rho + 12) \log \left( \frac{\rho}{2} \right) + (\rho^2 - 6\rho + 12)e^{\rho}\Gamma[0, \rho] \right),
\] (13c)

where \( \Gamma[0, \rho] \) is the incomplete gamma function [4]. We remark that it was simpler to re-derive \( H_1 \) in expression (11) for Stokes flow rather than letting \( Sc \to \infty \) in the solutions in [2].

### 2.1. The outer solution \( H_2 \)

We have derived the solution for the next term, \( H_2 \), in the outer expansion to be:

\[
H_2 = e^{\beta\rho\mu} \left( \frac{\beta}{1 + \beta} \right) \left[ \left( \frac{\pi}{\rho} \right)^{\frac{1}{2}} \sum_{k=0}^{4} A_2^{(k)} K_{k+\frac{1}{2}} \left( \frac{\rho}{2} \right) P_k (\mu) + \sum_{k=0}^{4} \beta_{2}^{(k)} (\rho) P_k (\mu) \right].
\] (14)

Matching \( \sum_{n=0}^{2} \text{Pe}^{n+1} H_n \) to \( \sum_{n=0}^{2} \text{Pe}^{n} h_n \) using van Dyke’s rule determined the constants \( A_2^{(k)} \):

\[
A_2^{(0)} = \frac{\beta(\log \text{Pe} + \gamma)}{2\pi(\beta + 1)} - \frac{419\beta^3 + 2172\beta^2 + 3253\beta + 1260}{960\pi(\beta + 1)^2(\beta + 2)}, \quad A_2^{(1)} = \frac{3(\beta + 3)}{16\pi(\beta + 2)}. \] (15)

The functions \( G_2^{(k)}(\rho) \) in expression (14) are given by:

\[
G_2^{(k)}(\rho) = -\frac{1}{\rho^2} K_{k+\frac{1}{2}} \left( \frac{\rho}{2} \right) \int_{\rho}^{\infty} u^{\frac{k}{2}} I_{k+\frac{1}{2}} \left( \frac{u}{2} \right) R_k (u) \, du
\]

\[
- \frac{1}{\rho^2} I_{k+\frac{1}{2}} \left( \frac{\rho}{2} \right) \int_{\rho}^{\infty} u^{\frac{k}{2}} K_{k+\frac{1}{2}} \left( \frac{u}{2} \right) R_k (u) \, du,
\] (16)

where

\[
R_0(\rho) = \frac{9e^{-\frac{\rho}{2}}}{20\rho^4} \left\{ 2\rho + \frac{9 - \beta}{18(\beta + 1)} \rho^2 - (\rho^2 + \rho + 2) (\log \rho + \gamma) - (\rho^2 - \rho + 2) e^{\rho}\Gamma(0, \rho) \right\},
\] (17a)

\[
R_1(\rho) = \frac{9e^{-\frac{\rho}{2}}}{40\rho^5} \left\{ -12\rho + \frac{\beta - 9}{3(\beta + 1)} \rho^2 - \frac{4\beta + 9}{3(\beta + 1)} \rho^3 + (4\beta^3 + 9\rho^2 + 6\rho + 12) (\log \rho + \gamma) - (4\beta^3 - 9\rho^2 + 6\rho - 12) e^{\rho}\Gamma(0, \rho) \right\},
\] (17b)
where the functions \( h_2 \).

2.2. The inner solution \( h_3 \)

The next term in the inner solution, \( h_3 \), is then given by:

\[
R_2(\rho) = \frac{9e^{-\frac{\rho}{56\rho^2}}}{56\rho^4} \left\{ 27\rho + \frac{38\beta + 45}{9(\beta + 1)} \rho^2 - \left( 4\rho^2 + 17\rho + 34 \right) \log(\rho + \gamma) - \left( 4\rho^2 - 17\rho + 34 \right) e^\rho \Gamma(0, \rho) \right\}, \quad (17c)
\]

\[
R_3(\rho) = \frac{9e^{-\frac{\rho}{40\rho^2}}}{40\rho^4} \left\{ -48\rho - 12\rho^2 - 2\rho^3 + \left( \rho^3 + 6\rho^2 + 24\rho + 48 \right) \log(\rho + \gamma) - \left( \rho^3 - 6\rho^2 + 24\rho - 48 \right) e^\rho \Gamma(0, \rho) \right\}, \quad (17d)
\]

\[
R_4(\rho) = \frac{9e^{-\frac{\rho}{280\rho^4}}}{280\rho^4} \left\{ 12\rho + 3\rho^2 - \left( \rho^2 + 6\rho + 12 \right) \log(\rho + \gamma) - \left( \rho^2 - 6\rho + 12 \right) e^\rho \Gamma(0, \rho) \right\}. \quad (17e)
\]

The inner solution, \( h_3 \), is then given by:

\[
h_3 = \frac{\beta}{1 + \beta} \sum_{k=0}^{\infty} a_k \left( \frac{\beta - \frac{1}{\beta}}{k + 1 + \beta} \right) P_k(\mu)
\]

\[
+ \left( \frac{\beta}{1 + \beta} \right) \left\{ \left( \frac{d h_3^{(i)}}{d r} - h_3^{(i)}(1) \right) \right\} r^{-k-1} + h_3^{(i)}(r) \right\} P_k(\mu), \quad (18)
\]

where the functions \( h_3^{(i)}(r) \) are the \( r \)-components of the particular solutions given by:

\[
h_3^{(0)}(r) = \frac{r^2}{24} + \frac{5\beta + 3r}{16(\beta + 1)} \log r + \frac{19\beta^2 + 23\beta - 12}{120(\beta + 1)(\beta + 2)r^2} + \frac{\beta}{96(\beta + 1)r^3} - \frac{7\beta^2 + 9\beta - 4}{384(\beta + 1)(\beta + 2)r^2} + \frac{\beta}{480(\beta + 1)r^5}, \quad (19a)
\]

\[
h_3^{(1)}(r) = \frac{\beta}{16(\beta + 1)r^3} + \frac{\beta}{40} + \frac{\beta}{8} + \frac{\beta}{16r^3} - \frac{3}{8r} \left( \frac{1}{16r^3} - \frac{3}{8r} + \frac{1}{4} \right) \log r
\]

\[
- \frac{1920(\beta + 1)^2(\beta + 2)}{1920(\beta + 1)^2(\beta + 2)} + \frac{551\beta^3 + 2844\beta^2 + 4201\beta + 1668}{1280(\beta + 1)^2(\beta + 2)r^2}
\]

\[
+ \frac{23\beta + 1}{1920(\beta + 1)r^2} + \frac{5237\beta^4 + 42471\beta^3 + 118927\beta^2 + 132033\beta + 46980}{53760(\beta + 1)^2(\beta + 2)(\beta + 3)r^3}
\]

\[
- \frac{3(\beta + 23)}{8960(\beta + 2)r^4} + \frac{235\beta^2 + 1463\beta + 2238}{26880(\beta + 2)(\beta + 3)r^5} + \frac{3(\beta + 3)}{1280(\beta + 2)r^6} - \frac{9}{17920r^7}. \quad (19b)
\]

\[
h_3^{(2)}(r) = \frac{\beta}{\beta + 1} \left( \frac{\log r}{120} + \frac{(\beta - 3)r}{96\beta} - \frac{5}{48} + \frac{25\beta^2 + 42\beta - 4}{96\beta(\beta + 2)r} \right)
\]
Matching \( \sum_{n=0}^{3} \text{Pe}^n h_n \) to \( \sum_{n=0}^{2} \text{Pe}^{n+1} H_n \) using van Dyke’s rule determines the constants \( a_{3}^{(k)} \) to be:

\[
\begin{align*}
    a_{3}^{(0)} &= -\frac{\beta(2 \log \text{Pe} + \gamma)}{4(\beta + 1)} - \frac{9 \mathcal{I}_1}{20} - \frac{9(\beta - 1) \mathcal{I}_2}{40(1 + \beta)} - \frac{7793 \beta^3 + 3252 \beta^2 + 3973 \beta + 1260}{1920(\beta + 1)^2(\beta + 2)} , \\
    a_{3}^{(1)} &= -\frac{3 \log \text{Pe}}{8} + \frac{(6 \beta + 11)}{40(1 + \beta)} - \frac{3 \gamma}{8} , \\
    a_{3}^{(2)} &= -\frac{1}{24} .
\end{align*}
\]

In (20a), the constants \( \mathcal{I}_1 \approx -1.1063 \) and \( \mathcal{I}_2 \approx -0.5772 \) are calculated from the integrals:

\[
\begin{align*}
    \mathcal{I}_1 &= \int_{0}^{1} \frac{e^{-u}}{u^3} \left( 2u - (2 + u + u^2)(\gamma + \log u) - (u^2 - u + 2)e^{u} \Gamma(0, u) \right) \, du + \frac{1}{2u} \\
    &\quad + \int_{1}^{\infty} \frac{e^{-u}}{u^3} \left( 2u - (2 + u + u^2)(\gamma + \log u) - (u^2 - u + 2)e^{u} \Gamma(0, u) \right) \, du , \\
    \mathcal{I}_2 &= \int_{0}^{1} \left( \frac{e^{-u} - 1}{u} \right) \, du + \int_{1}^{\infty} \frac{e^{-u}}{u} \, du .
\end{align*}
\]

3. Results and discussion

The average Nusselt (Sherwood) number is an important measure of the rate of heat (mass) transfer from the surface of the sphere. It is defined by:

\[
\text{Nu} = -\frac{1}{2} \int_{-1}^{1} \frac{\partial h}{\partial r} \bigg|_{r=1} \, d\mu ,
\]

and is equal to (cf. Gupalo and Ryazantsev [2] in the limit that \( \text{Se} \to \infty \)):

\[
\text{Nu} = \frac{\beta}{1 + \beta} \left( 1 + \frac{\beta}{2(1 + \beta)} \left( \text{Pe} + \text{Pe}^2 \log \text{Pe} \right) + \frac{\beta}{1 + \beta} \left( \frac{\gamma}{2} + \frac{121 \beta^2 + 168 \beta - 193}{960(\beta + 1)(\beta + 2)} \right) \text{Pe}^2 \\
+ \frac{\beta^2}{2(1 + \beta)^2} \text{Pe}^3 \log \text{Pe} + \frac{\beta}{1 + \beta} \left( \frac{9 \mathcal{I}_1}{20} + \frac{(9 - \beta) \mathcal{I}_2}{40(1 + \beta)} + \frac{\gamma \beta}{4(\beta + 1)} \right) \right) \text{Pe}^3 \\
- \frac{359 \beta^4 + 1692 \beta^2 + 2353 \beta + 900}{960(\beta + 1)^2(\beta + 2)} + o(\text{Pe}^3) .
\]
Figure 1: Plot of \((1 + \beta \overline{\text{Nu}}/\beta - 1)/\text{Pe}\) as a function of the Péclet number, 0.01 \(\leq \text{Pe} \leq 0.4\), with \(\beta = 1\) calculated analytically from (23) (solid line) and numerically (dot-dashed line). The dashed line is the previous asymptotic approximation derived by Gupalo and Ryazantsev [2] (in the limit \(\text{Sc} \to \infty\)).

Letting \(\beta \to \infty\) in (23) corresponds to constant temperature (concentration) on the sphere, so that \(h = 1\) on \(r = 1\), (cf. Acrivos and Taylor [1]), and gives:

\[
\overline{\text{Nu}} = 1 + \frac{1}{2} \text{Pe} + \frac{1}{2} \text{Pe}^2 \log \text{Pe} + \text{Pe}^2 \left(\frac{\gamma}{2} + \frac{121}{960}\right) + \frac{1}{2} \text{Pe}^3 \log \text{Pe}
\]

\[
+ \text{Pe}^3 \left\{\frac{\gamma}{4} - \frac{359}{960} + \frac{9 Z_1}{20} - \frac{Z_2}{40}\right\} + o(\text{Pe}^3). \tag{24}
\]

In both expressions, (23) and (24), the coefficient of \(\text{Pe}^3 \log \text{Pe}\) is twice that reported in [1] and [2]. Figure 1 shows a comparison of the analytical solution, (23), to numerical simulations for \(\beta = 1\) as a function of \(\text{Pe}\). As expected, this plot shows that the new asymptotic solution (23) is valid over a substantially increased range of Péclet numbers; the percentage error in the \(O(\text{Pe})\) and higher order contributions to \((1 + \beta)\overline{\text{Nu}}/\beta\) is within 8% for \(\text{Pe}\) up to 0.3.

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