“If I remember rightly, $\cos\frac{\pi}{2} = 1$”

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Introduction

Some time ago, I returned from a year abroad to find a scrap of paper pinned on the otherwise empty noticeboard in my new office. It said “C is bounded with an unbounded bound”. The words had been spoken by the previous occupant during a spirited argument about something or other, now long forgotten, and had fortunately been recorded. Absurdities like this are more common than one might suppose, and having started out listening for them, the attuned ear soon amasses a fair collection (a similar position is seen with Spoonerisms [1]). In Britain they are called Colemanballs, a reference to sports commentator David Coleman whose remarks, along the lines of “and for those of you watching who don’t have television sets, there is live commentary on Radio 2”, gave rise to a regular column of ‘Colemanballs’ collected by readers of the satirical magazine Private Eye [2]. In the US I have heard them referred to as ‘jokes’, but this seems to me to miss the point. A classic Colemanball is above all unintentional; if the absurdity is compounded by hubris, as in the example of the title, which I recall was delivered in a particularly superior tone of voice, then so much the better. Protestation by the perpetrator that “you know perfectly well what I meant” (as is indeed sometimes the case) may properly be ignored.

Applied mathematics, and in particular the informal and mildly competitive discussions of workshop or Study Group activities such as those held at Oxford or RPI, has been a fruitful source of Colemanballs. Perhaps this is because applied mathematicians are not overly concerned with rigour. Perhaps also many Colemanballs originate at places like infinity which have no real-world counterpart, and hence tax our skills of translating the abstract into the concrete. Perhaps we just don’t think before we speak. . . . At any rate a good Colemanball has an unmistakable ring that is not exactly truth, more artistic verisimilitude.

What follows is a selection from the Blue Book, kept in the Common Room of the Oxford Centre for Industrial and Applied Mathematics. In it, we record the Colemanballs collected by our spies (who are everywhere); they are set down verbatim - improvements are forbidden. Readers interested in a similar collection, this time of comments made by lecturers at Cambridge University, should see [3]; it includes the classic "Damn, I'm running out of integers".

Zero and Infinity

Applied mathematicians clearly have trouble with the concept of infinity (pure mathematicians probably do too, although they will not admit to it; see [4] pp 137-8). Indeed, I suspect that most simply think of it as being rather a long way off the right-hand side of the page. Surely this was in the mind of the colleague who decided that "you can always make infinity smaller by multiplying by h", or that "the stagnation point might go half-way to infinity and then stop". Further evidence for the finiteness of infinity is "the velocity [which] is much bigger than infinity", although "in this case you can take infinity to be quite small". Zero is generally better understood, being nice and central in the real line, but spare a thought for the topology behind "it's infinite there, even though it's zero".

Large and small

Even finite numbers can be confusing: "What size is the Peclet number?"; "It's 10^6"; "Now remind me, is that big or small?". For some reason, small numbers seem to hold more terrors than large ones, especially when encountered as orders of magnitude, as in "in that frame of reference, $O(1)$ is transcendentally small", or "it's not fair to look at a small point". Sometimes the two are confused: the colleague who said "instead of considering 10 as large, let's consider 10 as small" might well envy the certitude of judgment of "there are some big ones and some very infinitesimal ones", or of "the exponential term is large – in fact it's exponentially large".

Geometry

There are some mighty strange shapes in the Colemanballs geometry. The "long thin spherical region" is quite common (in two dimensions, "Chapter 3 deals with the thin circular geometry"), as is "the sunfish [which] is shaped like a nearly spherical disc" (maybe also "like a triangular disc", which has the property that "one side of a triangle is always shorter than the sum of the other three sides"). Curves and lines, too, are unusual: "a hyperbola has a straight bit", while "the straight line has by its nature changed its curvature at each end". The latter is quite unrelated to "a parabola with fixed curvature". Even rigid bodies can move in unexpected ways, like the "rigid pulsating cylinder", which doubtless has a “rigid flexible boundary”. At least “a circle’s isotropic – it looks the same from every direction”.

Count-ability

‘There are three kinds of mathematician, those who can count and those who can’t.’ This was a joke, but deadly serious was the statement that “the only time you get interesting behaviour is when a is $O(1)$, or very small, or very large, or one of the two”, and so was “there are 4 quantities: 3 are the same and one of the others is different”. Also in the latter class were the colleagues who averred “there’s only one word for that: ‘free boundary problem’”, and those who observed “that’s interesting: the bubbles rise in pairs of three - no, four”, and “I think the coefficients always add up to 2, so 1, 2 and 1 will do it”.

Logic

Logic is not the forte of Colemanballs mathematicians, whose seminar titles are along the lines of “Why are rarely-occurring phenomena not uncommon? A logician’s viewpoint”. At least “you can’t have $s = 0$ if it’s not equal to zero, as it were” was correct, but what could “if 10 corresponds to 23, then 23 is half of 45” have been intended to mean ? The question “how unique is that solution?” can be answered positively by “there is only one unique solution”, but perhaps it is best in the end to admit that “I have only reached inconclusive conclusions”.

Physics

Colemanballs physics is a topsy-turvy subject. “The solid mass of injected liquid” may have a “stress tensor [which] is completely isotropic; the only nonzero element is σ_{23}”, although “gas doesn’t have a pressure in its own right”. “Heat has a perfect thermally insulating role”, or, even weirder, “you’ve got to have some cold heat”, but even with cold heat, “the temperature at this point never gets above the maximum temperature”. The system of units in which “I’m thinking of a time $t^{1/2}$ and a distance t downstream”, “which gives a length of 10 seconds”, and, conversely, “the thermal timescale . . . is about 25 metres”, is also that in which “the mass of this thing is about 1 kilometre”. Although “nobody’s interested in measuring things that can’t be measured”, “they measure time chronologically”, perhaps to solve a “problem [which] is unsteady in time”. “When you lean forward, gravity changes”, so that “there is a horizontal gravity component”; if you do this often enough, “gravity acts in several directions”.

Nonlinearity

We all prefer to avoid nonlinear problems if possible[1] ("$dy/dx = e^x$ ...looks pretty nonlinear to me", in a suspicious tone of voice), so although "when the nonlinear terms in an equation occur at the same order as the linear terms, and we find the lowest order term in an asymptotic expansion, the resulting equation is nonlinear", the usual result is that "it's linear except for the nonlinear terms". The resulting problem may be "a nonlinear Burgers equation" and may have "a spectrum that is almost continuous except that it's discrete", with "eigenvalues crossing the imaginary axis at smaller and smaller values of i".

Singularities

The best way to deal with these is probably to "regularize the problem by removing the term causing the singularity"; this would be sensible if "the singularity is latent in the branch point" or if you "don't like non-analytic singularities".

A rigorous result

Theorem: Bearing in mind the fact that "it is not unknown for things not to tend to their limit", it is possible to "approximate an irrational point by a sequence of increasingly irrational rationals".
Proof: "I have an empirical proof of this theorem".

Mathematical technicalities

Nothing simpler than a periodic function? Well, try the equation with "periodic solutions within one period" (during which "the phase changes by 180° and the amplitude changes sign") or "the function $g(x)$ which is slightly periodic". Still on a technical note ("$x > 0$: these sorts of issues really need to be put in mathematical terms"), we see that "of course, in regions where the yield stress is not exceeded, the stress is indeterminate (and perhaps not a particularly useful concept), but it may be assumed differentiable with respect to x"; or at least it may be "slightly differentiable".

Constants

Colemanballs constants are not without interest ("if you've taken them arbitrary, you've taken them to be something"). Leaving aside the "small $O(1)$ constant", and the "constants [which] are either zero or not zero", we have "arbitrary constants which can be functions of space and time" (except "in the present situation [where] the steady–state solution is *truly* time–independent")

[1] We except the valiant but misguided "you can homogenize it but you don't get a nice nonlinear thing".

and the nonconstant constant, as in "the viscosity is constant but varying". These shifting sands perhaps provoked the assertion "it's constant for all time" and the immediate riposte "what, the same constant?". Maybe the variable constant is really the mythical $c(t)$ occurring in "we can solve the problem and find $c(t)$ even though $c(t)$ isn't in the problem".

Conclusion

Some of the assertions presented above may be hard to believe ("there are critical criticisms of this speculation"), and some of the methodology suspect ("there is a word for that beginning with s: heuristic"). We conclude by reassuring the reader of the verities that "classical slender body theory has in mind the slender body", and that "at the end of the day, the best we can hope for is $1 = 1$".

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